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Electroweak Sphaleron in Large Higgs Boson Mass Limit

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ABSTRACT

Since the triviality argument of the Higgs sector requires the existence of new physics beyond the standard model, there should exist a cutoff Λ beyond which the standard model will breakdown. The cutoff can be determined from the position of the Landau pole. We study the effect of this cutoff on the energy of the electroweak sphaleron, E^{spha} . We found that E^{spha} becomes arbitrarily large as the Higgs boson mass increases, caused by the existence of a dimension eight operator. This is in contrast to the well-known result, which is held in the standard model and a wide class of its extensions, that E^{spha} is stable against the variation of the Higgs boson mass. The physical meaning of this result is discussed.

1. The sphaleron in the electroweak theory plays an essential role in the calculation of the baryon number violation in colliders and at high temperature, because it sets the energy scale at which the nontrivial structure of the finite energy configuration space becomes apparent. More concretely, the sphaleron gives the minimal height of the energy barrier between the topologically inequivalent neighbouring vacua and this barrier determines the rate of the (B+L), the baryon number plus lepton number, violating reactions. The energy of the sphaleron in the standard model (SM), calculated first by Manton and Klinkhamer in a spontaneously broken SU(2) theory [1], then improved in [2], is given by $E^{spha} = \frac{2M_W}{\alpha_W} B(\lambda/g_W^2)$, where M_W is the W-boson mass, $\alpha_W = g_W^2/4\pi$ and g_W is the SU(2) coupling constant; B depends on the ratio of the Higgs quartic coupling, λ , and g_W . Numerical evaluation gives $1.5 \leq B \leq 2.7$ when the *bare* Higgs boson mass, $m_H^2 = 2\lambda v^2$, varies from zero to infinity[F.1]. Hence the value of E^{spha} is extremely stable against the change in the Higgs boson mass. Furthermore, the stability of the sphaleron energy against the changes of models for a wide class of models has been demonstrated. The models considered include the extensions of the standard one-doublet Higgs sector and the addition of dimension six operators to the standard model lagragian[4]. Specifically, the values of E^{spha} in those extended models have been shown to change very little for a given Higgs boson mass, much less than the variation of its standard model value due to the change of the Higgs boson mass.

Based on the well-known triviality argument of the Higgs sector of the standard model, it is often argued that the standard model is an effective theory and there must be a more fundamental theory beyond SM. In particular, the Higgs sector of the standard model will have to be modified in this case. The scale of the new physics, parametrized in terms of a cut-off Λ , beyond which SM has to be modified, can be estimated by the location of the Landau pole of the Higgs quartic

coupling constant. Approximately one has[5],

$$\Lambda \leq v \exp(\frac{8\pi^2 v^2}{3m_H^2}) = v \exp(\frac{2\pi}{\alpha_W(\lambda/g_W^2)}) \quad , \quad (1)$$

where v is the Higgs vacuum expectation value and m_H the Higgs boson mass. One can see that as Higgs boson mass increases, the cut-off Λ decreases and the new physics effects become more important near the electroweak symmetry breaking scale v . In this paper, we study the sensitivity of the SM sphaleron energy to new physics required by the consideration of the “triviality” of the SM Higgs sector.

The importance of the effect of new physics in the energy regime below the electroweak energy scale in relation to the cutoff has been studied by Corteses, Pallante and Petronzio[5] in the context of the standard model electroweak radiative correction to the LEP data due to Heavy Higgs. They concluded that the decreasing of the location of the Landau pole with increasing Higgs boson mass implies that the sensitivity to the effect of the radiative correction to the value of the Higgs boson mass for heavy Higgs will be lost due to the sizable cut-off dependence of the theory as Λ approaches v for very large m_H . Generally, the cut-off effects can be accounted for by adding higher dimension operators to the standard model lagrangian. In this paper we demonstrate that the effect of the cut-off can cause E^{spha} to increase without bound as Higgs boson mass goes to infinity.

2. Below we briefly review the SM sphaleron[F.2]. Using the spherical symmetric ansatz[1], we can write the static fields in the $W_0 = 0$ gauge,

$$W_i^a \sigma^a dx^i = -\frac{2i}{g_W} f(\xi) dU^\infty (U^\infty)^{-1} \quad ; \quad (2.a)$$

$$\phi = \frac{v}{\sqrt{2}} h(\xi) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad , \quad (2.b)$$

where

$$U^\infty = \frac{1}{r} \begin{pmatrix} z & x + iy \\ -x + iy & z \end{pmatrix} ,$$

and $\xi = g_W v r$.

The energy functional is given by

$$E^{spha} = \int d^3x \left[\frac{1}{4} F_{ij}^a F^{aij} + |D_i \phi|^2 + \lambda(|\phi|^2 - v^2/2)^2 \right] , \quad (3)$$

where

$$F_{ij}^a = \partial_i W_j^a - \partial_j W_i^a + g_W \epsilon^{abc} W_i^b W_j^c ;$$

$$D_i \phi = \partial_i \phi - \frac{i}{2} g_W W_i^a \sigma^a \phi .$$

The sphaleron energy is obtained by minimizing eq.(3). This gives rise to a set of coupled non-linear differential equations involving $f(\xi)$ and $h(\xi)$. Their solutions determine the sphaleron energy from (3). The boundary conditions for $f(\xi)$ and $h(\xi)$ are given by[1]

$$f(\xi) \rightarrow \xi^2 \quad \text{and} \quad h(\xi) \rightarrow \xi \quad \text{for} \quad \xi \rightarrow 0 ; \quad (4.a)$$

$$f(\xi) \quad \text{and} \quad h(\xi) \rightarrow 1 \quad \text{for} \quad \xi \rightarrow \infty. \quad (4.b)$$

To estimate E^{spha} , let us use the Ansatz of Klinkhamer and Manton [1],

$$f(\xi) = \frac{\xi^2}{\frac{\Xi}{\Xi+4}}, \quad \text{for} \quad \xi \leq \Xi ; \quad (5.a)$$

$$f(\xi) = 1 - \frac{4}{\Xi+4} \exp\left[\frac{1}{2}(\Xi - \xi)\right], \quad \text{for} \quad \xi \geq \Xi , \quad (5.b)$$

and

$$h(\xi) = \frac{\sigma\Omega + 1}{\sigma\Omega + 2} \frac{\xi}{\Omega}, \quad \text{for } \xi \leq \Omega ; \quad (5.c)$$

$$h(\xi) = 1 - \frac{\Omega}{\sigma\Omega + 2} \frac{1}{\xi} \exp[\sigma(\Omega - \xi)], \quad \text{for } \xi \geq \Omega , \quad (5.d)$$

where Ξ and Ω are determined by minimizing the energy functional for a given values of λ/g_W^2 . Some of these values given in Klinkhamer and Manton [1] are listed in table I to show how Ξ and Ω varies with λ/g_W^2 .

λ/g_W^2	Ω	Ξ
0	2.600	2.660
10^{-3}	2.520	2.450
10^{-2}	2.290	2.120
10^{-1}	1.900	1.650
1	1.250	1.150
10	0.620	0.820
10^2	0.220	0.740
10^3	0.070	0.730
∞	0	0.728

Table I.

3. Now let's consider the cut-off effects on the SM sphaleron energy. We have examined the effect of many higher dimension operators which involve the Higgs field and are invariant under the standard model gauge symmetry. However, we will concentrate here on the following most interesting operator for detail discussion,

$$\mathcal{O} \sim \frac{1}{\Lambda^4} \{ (D_\mu \phi)^\dagger D^\mu \phi \}^2 . \quad (6)$$

The reason why we choose \mathcal{O} is that it is the operator of the lowest dimension that makes E^{spha} diverge in the heavy Higgs boson mass limit. We present our arguments below.

Using the spherical symmetric ansatz (2), we have the contribution of the operator \mathcal{O} to E^{spha} ,

$$\Delta E^{spha} = \frac{g_W \pi v^5}{\Lambda^4} \int d\xi \left\{ \xi^2 \left(\frac{dh}{d\xi} \right)^4 + 4h^4 \left(\frac{dh}{d\xi} \right)^2 (1-f)^2 + 4 \frac{h^4}{\xi^2} (1-f)^4 \right\} . \quad (7)$$

The last term in the right-handed side causes ΔE^{spha} to diverge as $\frac{\lambda}{g_W^2} \rightarrow \infty$. This can be seen clearly by applying the ansatz eqs.(5), although the result is independent of this particular ansatz. The last term of the right-handed side of eq.(7) dominates for very heavy Higgs,

$$\Delta E^{spha} \sim \int d\xi \frac{h^4}{\xi^2} (1-f)^4 \rightarrow \frac{1}{\Omega} . \quad (8)$$

From table I we see that $\Omega \rightarrow 0$ and hence $\Delta E^{spha} \rightarrow \infty$ as $\lambda \rightarrow \infty$.

The result is not a consequence of the "perturbative" treatment given above. Incorporating the operator \mathcal{O} directly into the sphaleron differential equations, and solving the differential equations without using the particular ansatz (5.a ~ d), our above conclusion still holds. The reason for the blowup of the sphaleron energy in the limit of very large Higgs boson mass is the following. The boundary conditions (4.a) and (4.b) are still valid with the addition of the new term (6) directly to (3). The corresponding differential equations for $f(\xi)$ and $h(\xi)$ are obtained by minimizing the modified energy expression. In the limit of large λ the Higgs potential term in (3) requires that $|\phi|^2 \rightarrow v^2/2$, (equivalently $h \rightarrow 1$) for $\xi > 0$, and $\phi = 0$ for $\xi = 0$. This implies that (8) diverges in the limit of very large λ or the Higgs boson mass.

4. To illustrate the physical meaning of the above result, we consider a toy model of the dynamical symmetry breaking (DSB) theory. Let us consider a one

family standard model where there is no elementary Higgs field. In this model the SM gauge symmetry is broken by the quark condensate driven by the QCD interaction, where both the electroweak symmetry breaking scale and the weak gauge boson masses are, of course, very low. Then the Higgs fields, σ and the Goldstone pions, are composites, made of the up and down quarks.

It is well-known that the strong interaction of a DSB model can produce various higher dimension operators when the heavy fermions, i.e., the quarks, are frozen out at energy below the DSB strong interaction scale. Integrating (freezing) out the heavy fermions will produce $SU(2)$ triplet and singlet effective operators which are arranged in some gauge invariant way. Then an operator such as the \mathcal{O} naturally exist[6]. Let us write $\phi = \frac{h}{\sqrt{2}}\Sigma$ with Σ being the unitary part of the Goldstone field and $h = H + v$, where H is the physical Higgs field. For the dynamical model, where no physical Higgs field appears in the effective lagrangian, operator \mathcal{O} is reduced to

$$\tilde{\mathcal{O}} \sim ((D_\mu \Sigma)^\dagger D^\mu \Sigma)^2 \quad , \quad (9)$$

which exists in the effective lagrangian of pion fields of the QCD.

Below the DSB strong interaction scale, there are only leptons in the fermion sector, and the lepton number current is violated by an $SU(2)$ anomaly which involves two $SU(2)$ currents. However, its amplitude will vanish according to our results since the Higgs boson mass in a composite Higgs theory is effectively infinite and hence E^{spha} is also infinity. Is this understandable? In the fundamental lagrangian of quarks and leptons, there are two kind of fermion number currents, one lepton number, and the other baryon number. Both have an $SU(2)$ anomaly, however, their difference is anomaly free, which means that the total change of the lepton number must equal to the amount of baryon number change. Since baryon

fields do not exist in the low energy lagrangian[F.3] lepton number violation process is forbidden. In other words, E^{spha} should be infinity.

Another example which is also an application of our result is the one-family technicolor model which has an electroweak sector similar to that of the ordinary, light fermions[8]. In this model, both the techni quarks and the ordinary quarks have an SU(2) anomaly. We can arrange the quantum numbers of the techni quarks such that the sum of the two baryon numbers is anomaly free. Below the technicolor scale, the physics is described by an effective theory where techni fermions are integrated out. The resultant effective lagrangian is similar, in part, to the meson effective lagrangian of QCD[9]. Therefore, higher dimension operators similar to $\tilde{\mathcal{O}}$ again appear. As it is argued above, E^{spha} should be infinity[F.4]. Therefore, in the scenario of such a model, baryon number violation at the electroweak scale is forbidden as the sphaleron energy is infinity. This recovers the fact that the overall baryon number, the sum of the techni and ordinary quarks, is conserved, even though only the ordinary fermions are the active degree of freedom in the low energy regime.

In conclusion, we took the standard model Higgs sector as an effective theory with higher dimension operators added, and calculated the correction of these higher dimension operators to the sphaleron energy. We found that the sphaleron energy diverges in the large Higgs boson mass limit. Without the higher dimension operators the sphaleron energy is stable against the variation of the Higgs boson mass. We have argued that there must be new physics above the Landau pole location and shown that the operator making sphaleron energy diverge does exist for example, in dynamical symmetry breaking models. And in those models, our result becomes obvious.

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Footnotes

- [F.1] The inclusion of the $U(1)_Y$, with the experimental value of the Weinberg angle, has very little effect on the sphaleron energy[3].
- [F.2] For simplicity, as usually we consider the limit of vanishing mixing angle Θ_W , so the $U(1)_Y$ field decouples.
- [F.3] One would expect solitons to exist at the vacuum expectation value scale which carry the baryon number[7]. However, these solitons could not be produced energetically at low energy. Furthermore, the soliton solutions are irrelevant to our discussions in this paper.
- [F.4] Integrating out a generation of heavy fermion, where the heavy fermion masses arise from Yukawa couplings, will give expressions such as $[\partial_\mu \phi \partial^\mu \phi]^2 / \phi^4$ [10, 11]. This term makes E^{spha} diverge for any value of the Higgs boson mass because of the ϕ^4 in the *denominator*, and the effective lagrangian is non-analytic. Recently, H. Georgi, L. Kaplan and D. Morin[11] pointed out that one must truncate the expansion of the lagrangian in powers of the *shifted* Physical Higgs field to get an well-defined effective lagrangian and argued that the instanton action in the effective theory is infinity due to the existence of a operator similar to $\tilde{\mathcal{O}}$. The operator \mathcal{O} in this paper is analytic and can make E^{spha} diverge only for infinity Higgs boson mass.

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